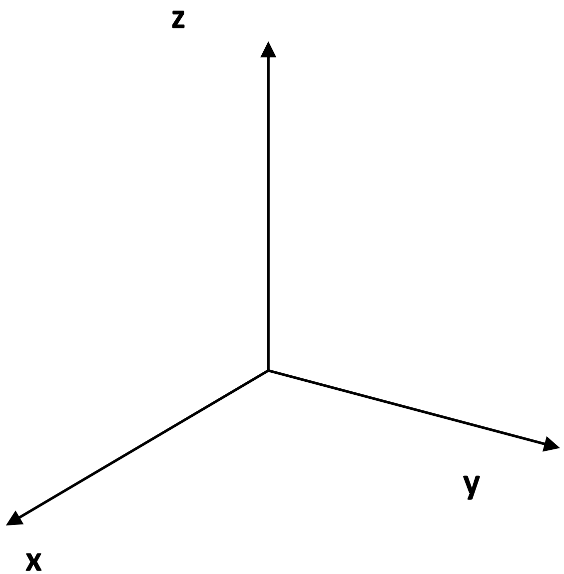
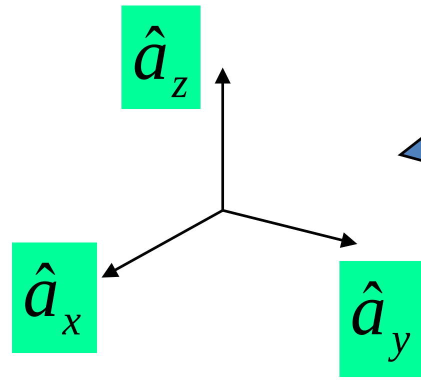
Lecture-3

Differential length, Area and volume

VECTOR REPRESENTATION: UNIT VECTORS

Rectangular Coordinate System





Unit Vector Representation for Rectangular Coordinate System

The Unit Vectors imply :

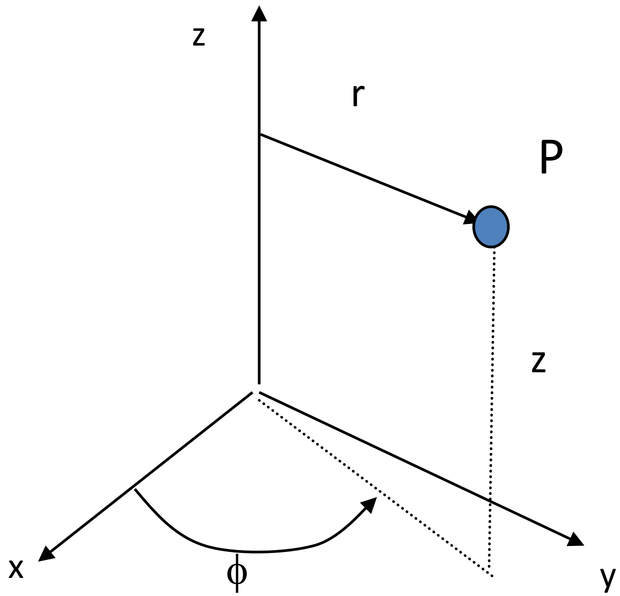
$\hat{a}_x$ → Points in the direction of increasing **x**

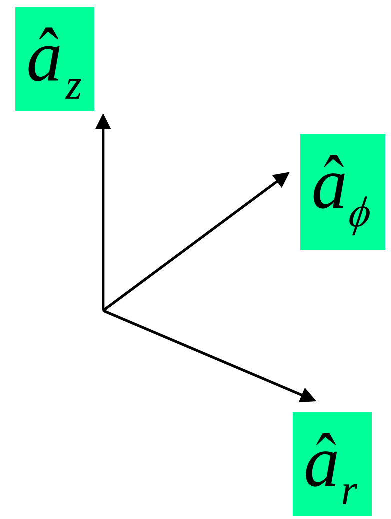
$\hat{a}_y$ → Points in the direction of increasing **y**

$\hat{a}_z$ → Points in the direction of increasing **z**

VECTOR REPRESENTATION: UNIT VECTORS

Cylindrical Coordinate System



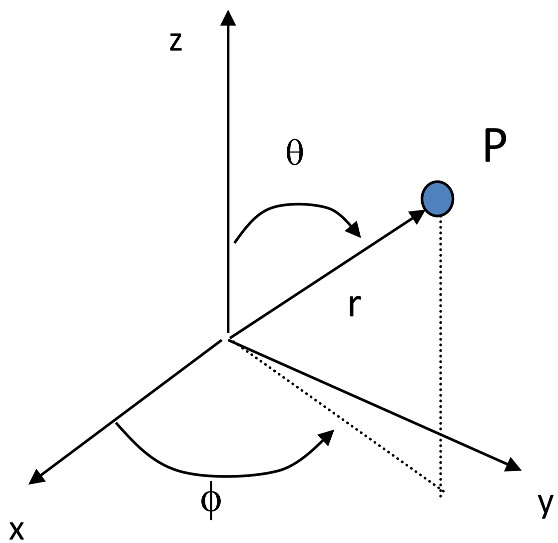


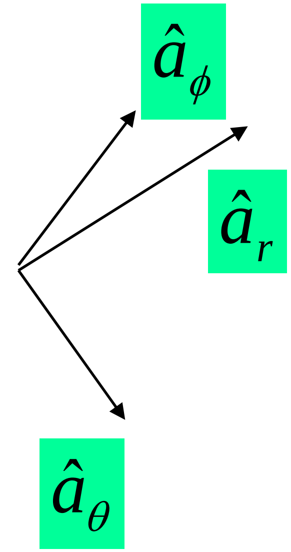
The Unit Vectors imply :

- $\hat{a}_r$ ➡ Points in the direction of increasing **r**
- $\hat{a}_\phi$ ➡ Points in the direction of increasing **φ**
- $\hat{a}_z$ ➡ Points in the direction of increasing **z**

VECTOR REPRESENTATION: UNIT VECTORS

Spherical Coordinate System





The Unit Vectors imply :

$\hat{a}_r$ ➡ Points in the direction of increasing **r**

$\hat{a}_\theta$ ➡ Points in the direction of increasing **θ**

$\hat{a}_\phi$ ➡ Points in the direction of increasing **φ**

VECTOR REPRESENTATION: UNIT VECTORS

Summary

RECTANGULAR Coordinate Systems	CYLINDRICAL Coordinate Systems	SPHERICAL Coordinate Systems
$\left(\hat{a}_x \quad \hat{a}_y \quad \hat{a}_z\right)$	$\left(\hat{a}_r \quad \hat{a}_\phi \quad \hat{a}_z\right)$	$\left(\hat{a}_r \quad \hat{a}_\theta \quad \hat{a}_\phi\right)$

NOTE THE ORDER!

r, ϕ, z $\qquad$ r, θ, ϕ

Note: We do not emphasize transformations between coordinate systems

METRIC COEFFICIENTS

Unit is in "meters"

1. Rectangular Coordinates:

When you move a small amount in **x**-direction, the distance is **dx**

In a similar fashion, you generate **dy** and **dz**

Cartesian Coordinates

Differential quantities:

Length:

$$d\vec{l} = \hat{x}dx + \hat{y}dy + \hat{z}dz$$

Area:

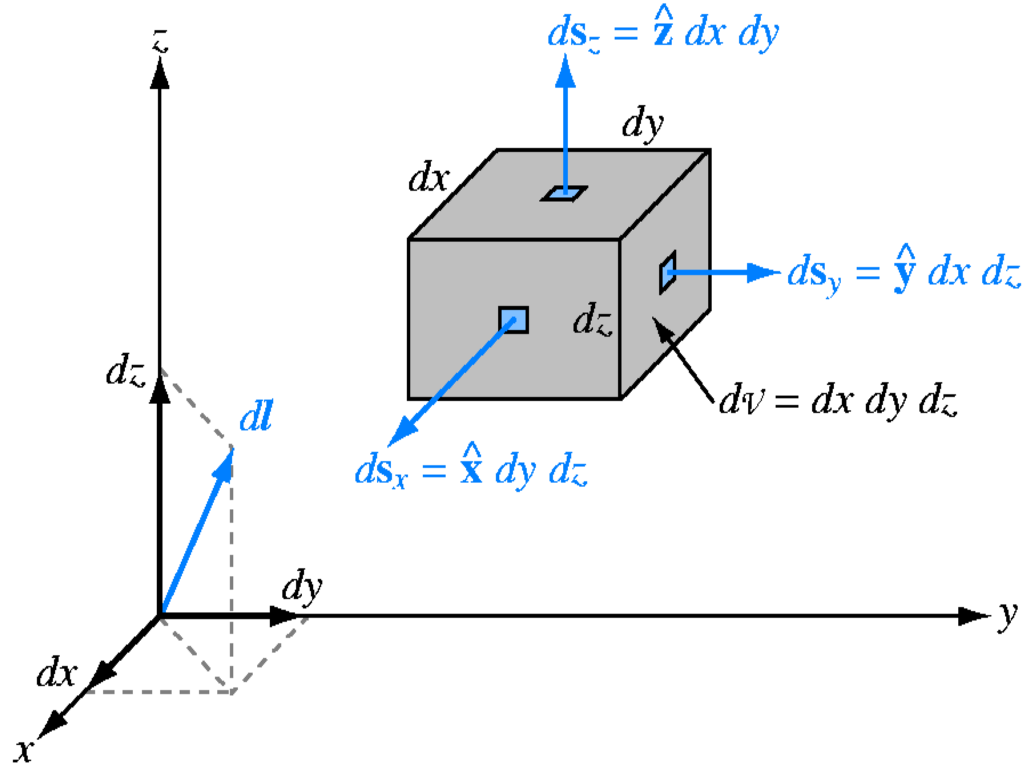
$$d\vec{s}_x = \hat{x}dydz$$

$$d\vec{s}_y = \hat{y}dxdz$$

$$d\vec{s}_z = \hat{z}dxdy$$

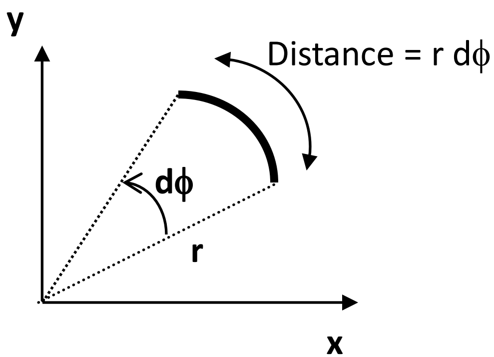
Volume:

$$dv = dxdydz$$



METRIC COEFFICIENTS

2. Cylindrical Coordinates:



Differential Distances:

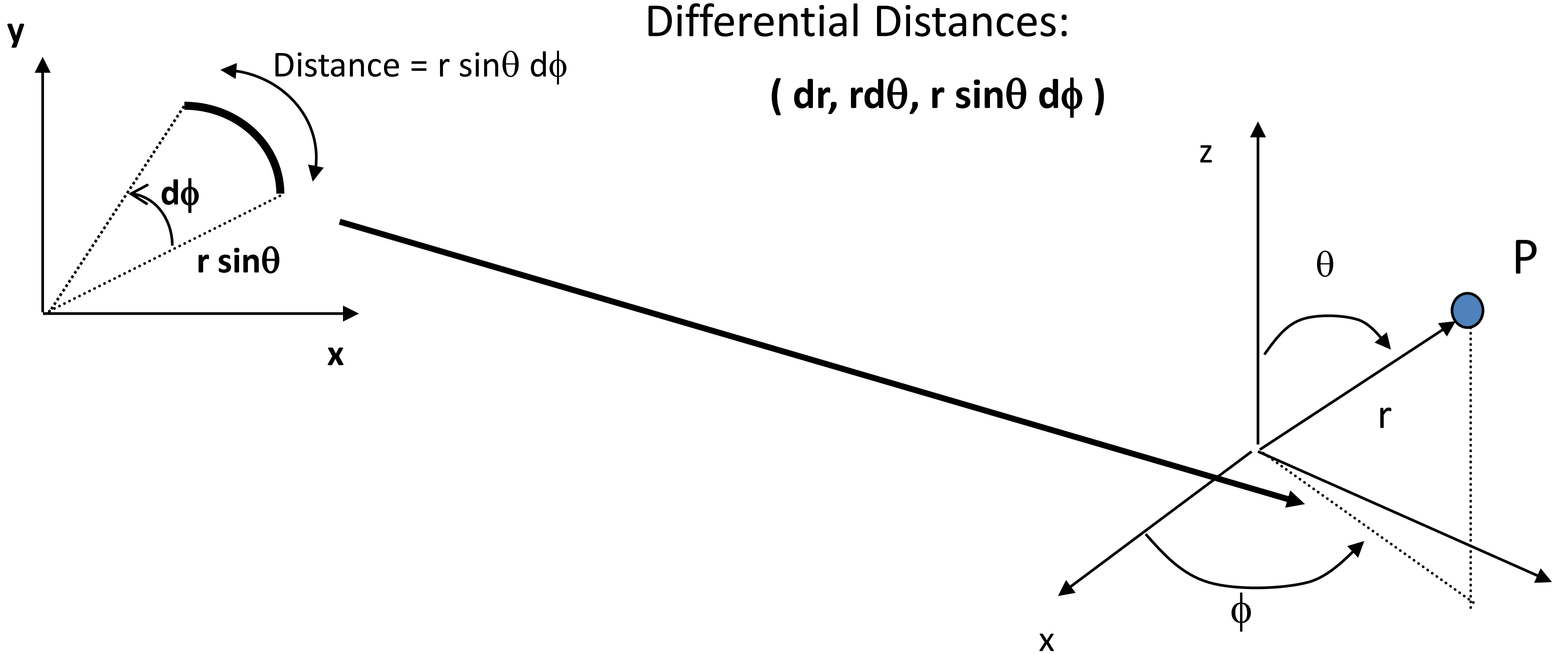
(dr, rdϕ, dz)

METRIC COEFFICIENTS

3. Spherical Coordinates:

Differential Distances:

(dr, rdθ, r sinθ dϕ)



METRIC COEFFICIENTS

Representation of differential length **dl** in coordinate systems:

rectangular

$$d\vec{l} = dx \bullet \hat{a}_x + dy \bullet \hat{a}_y + dz \bullet \hat{a}_z$$

cylindrical

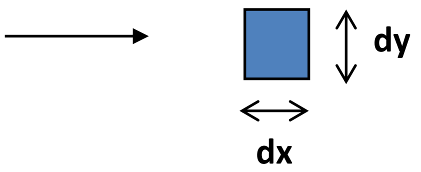
$$d\vec{l} = dr \bullet \hat{a}_r + r \bullet d\phi \bullet \hat{a}_\phi + dz \bullet \hat{a}_z$$

spherical

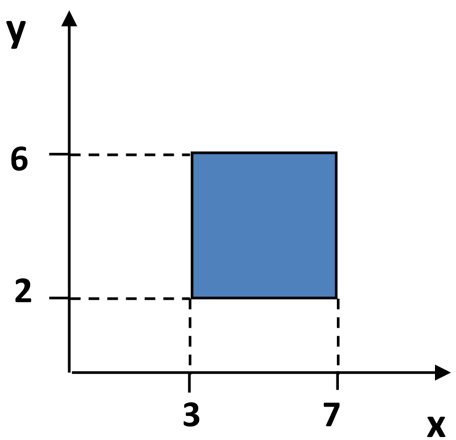
$$d\vec{l} = dr \bullet \hat{a}_r + rd\theta \bullet \hat{a}_\theta + r\sin\theta d\phi \bullet \hat{a}_\phi$$

AREA INTEGRALS

- integration over 2 "delta" distances



Example:



AREA = $\int_3^7 \int_2^6 dy \bullet dx$ = 16

Note that: ***z = constant***

In this course, area & surface integrals will be on similar types of surfaces e.g. **r =constant** or ϕ **= constant** or θ **= constant** et c….

SURFACE NORMAL

Representation of differential surface element:

Vector is NORMAL to surface

$$d\vec{s} = dx \bullet dy \bullet \hat{a}_z$$

DIFFERENTIALS FOR INTEGRALS

Example of Line differentials

$$d\vec{l} = dx \bullet \hat{a}_x$$ or $$d\vec{l} = dr \bullet \hat{a}_r$$ or $$d\vec{l} = rd\phi \bullet \hat{a}_\phi$$

Example of Surface differentials

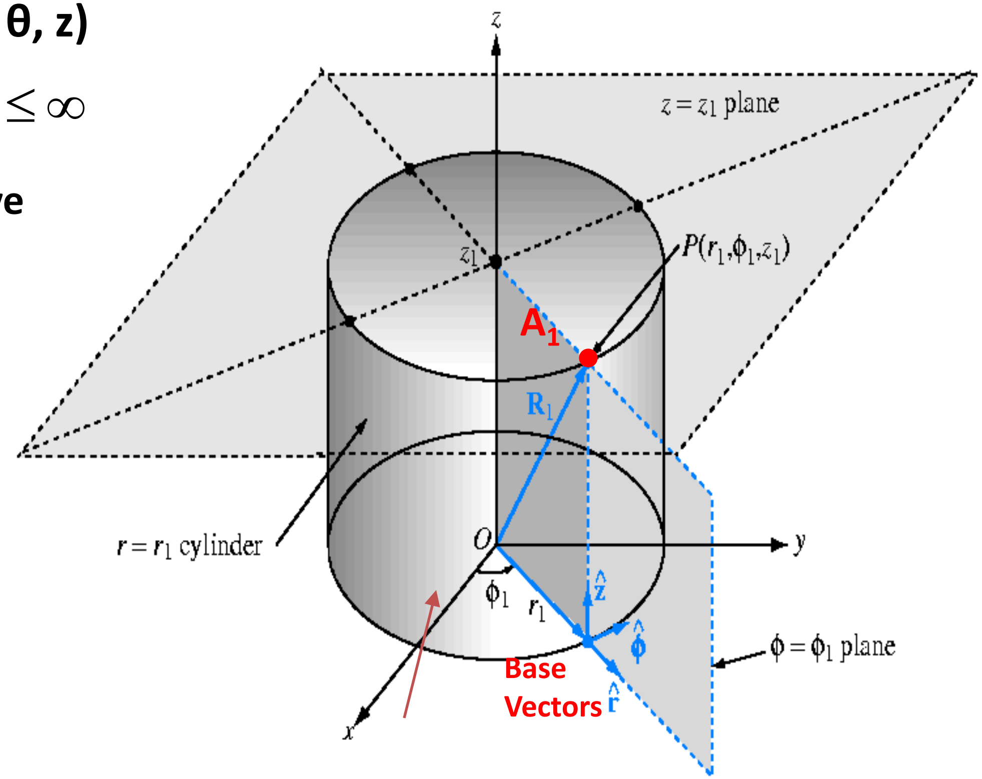
$$d\vec{s} = dx \bullet dy \bullet \hat{a}_z$$ or $$d\vec{s} = rd\phi \bullet dz \bullet \hat{a}_r$$

Example of Volume differentials ➡ $$dv = dx \bullet dy \bullet dz$$

Cylindrical Coordinates

(r, θ, z)

- **r radial distance in x-y plane** $0 \le r \le \infty$
- **Φ azimuth angle measured from the positive x-axis** $0 \le \Phi < 2\pi$
- **z** $-\infty < z < \infty$



Vector representation

$$\vec{A} = \hat{a}\left|\vec{A}\right| = \hat{r}A_r + \hat{\Phi}A_\Phi + \hat{z}A_z$$

Magnitude of A

$$\left|\vec{A}\right| = \sqrt[+]{\vec{A}\cdot\vec{A}} = \sqrt[+]{A_r^2 + A_\Phi^2 + A_z^2}$$

Position vector A

$$\hat{r}r_1 + \hat{z}z_1$$

Base vector properties

$$\hat{r}\times\hat{\Phi} = \hat{z},$$

$$\hat{\Phi}\times\hat{z} = \hat{r},$$

$$\hat{z}\times\hat{r} = \hat{\Phi}$$

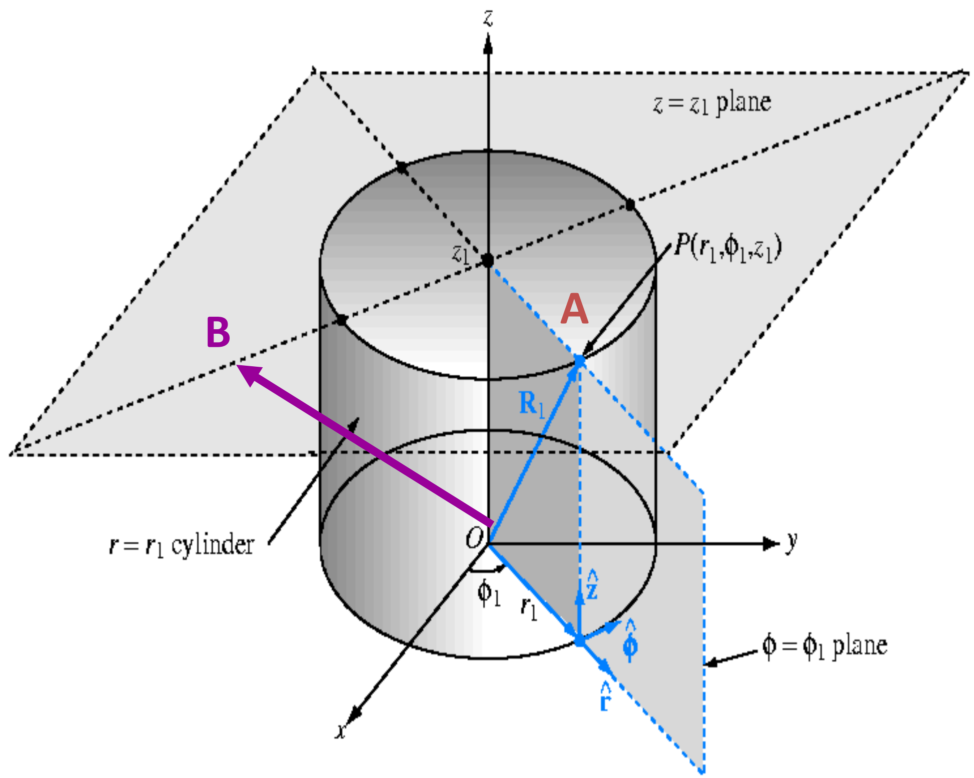
Cylindrical Coordinates

Dot product:

$$\vec{A}\cdot\vec{B} = A_r B_r + A_\phi B_\phi + A_z B_z$$

Cross product:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$$



Cylindrical Coordinates

Differential quantities:

Length:

$$d\vec{l} = \hat{r}dr + \hat{\Phi}rd\Phi + \hat{z}dz$$

Area:

$$d\vec{s}_r = \hat{r}rd\Phi dz$$

$$d\vec{s}_\Phi = \hat{\Phi}drdz$$

$$d\vec{s}_z = \hat{z}rdrd\Phi$$

Volume:

$$dv = rdrd\Phi dz$$

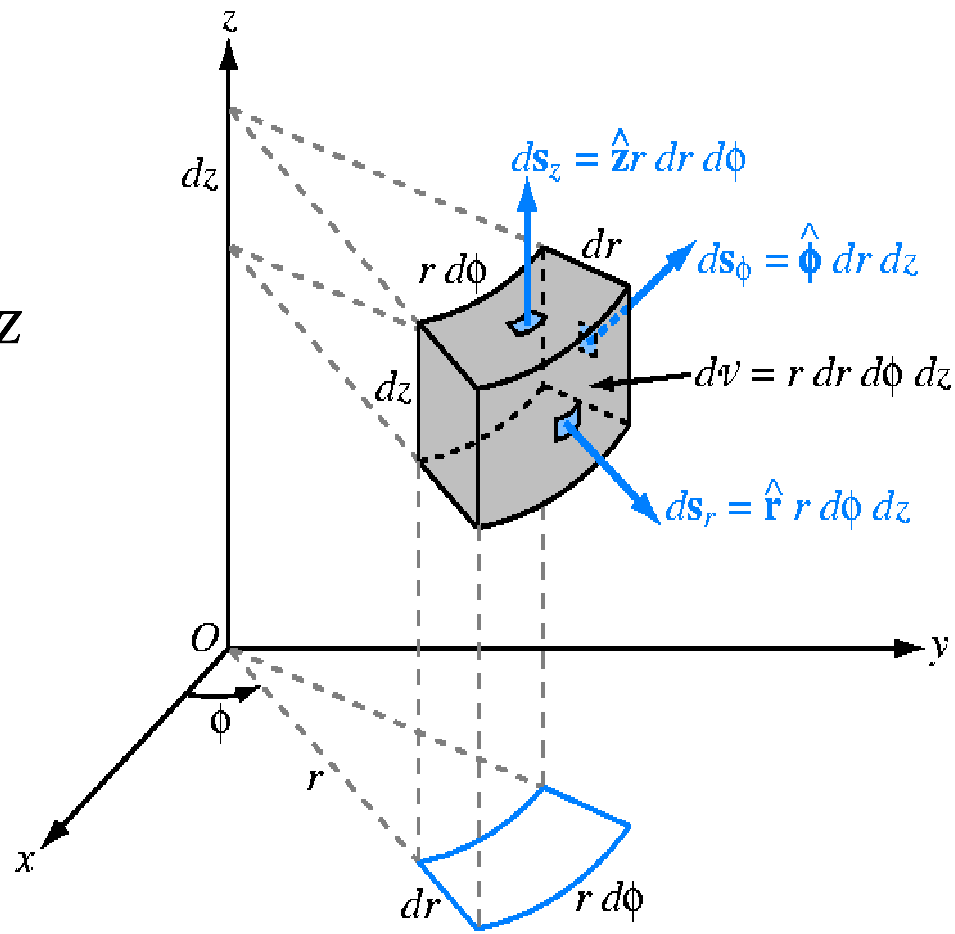


Figure 3-10

Spherical Coordinates

(R, θ, Φ)

Vector representation

$$\vec{A} = \hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$$

Magnitude of A

$$\left|\vec{A}\right| = \sqrt[+]{\vec{A}\cdot\vec{A}} = \sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$$

Position vector A

$$\hat{R}R_1$$

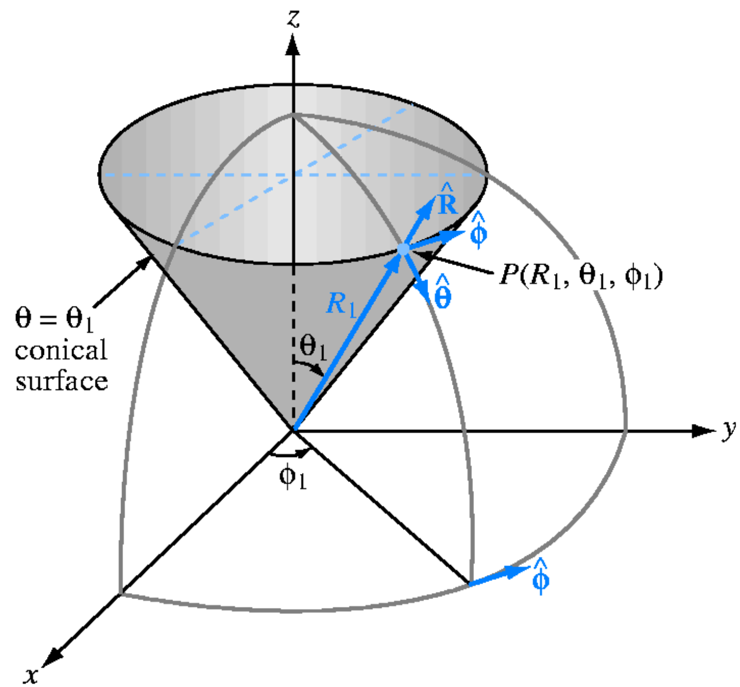


Figure 3-13

Pradeep Singla

Base vector properties

$$\hat{R}\times\hat{\Theta} = \hat{\Phi}, \qquad \hat{\Theta}\times\hat{\Phi} = \hat{R}, \qquad \hat{\Phi}\times\hat{R} = \hat{\Theta}$$

Spherical Coordinates

Pradeep Singla

Dot product:

$$\vec{A}\cdot\vec{B} = A_R B_R + A_\theta B_\theta + A_\phi B_\phi$$

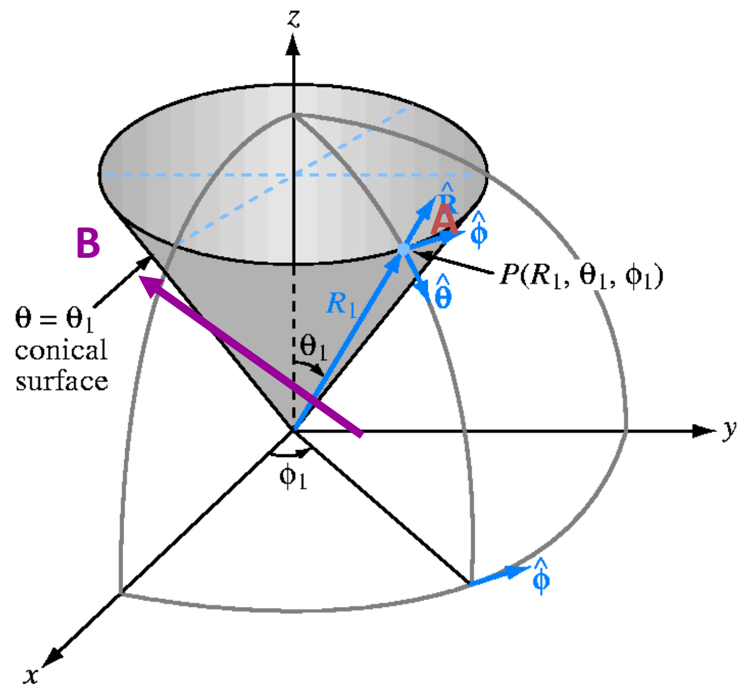


Figure 3-13

Cross product:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$$

Spherical Coordinates

Differential quantities:

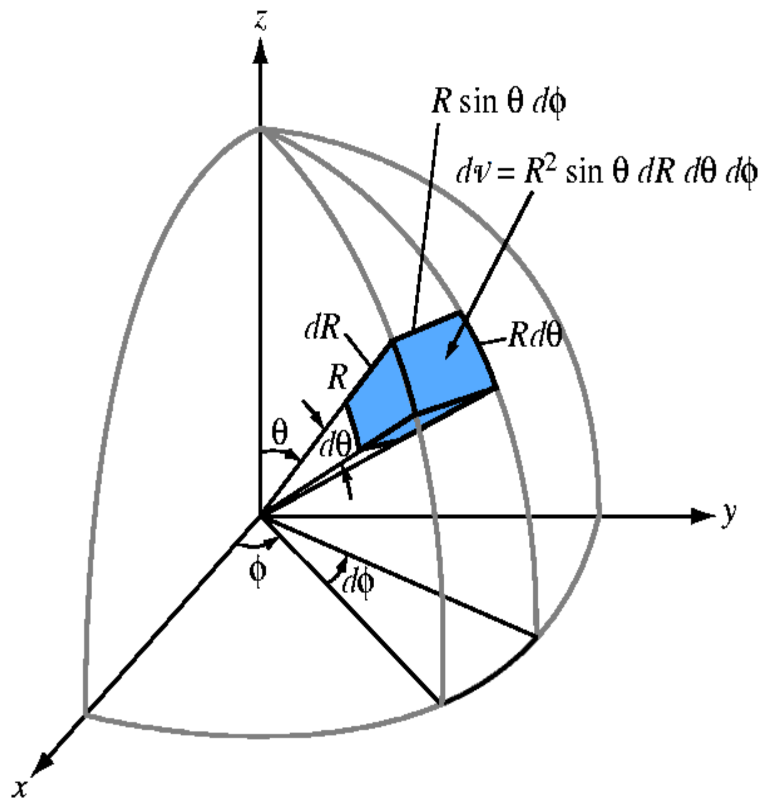
Length:

$$d\vec{l} = \hat{R}dl_R + \hat{\Theta}dl_\Theta + \hat{\Phi}dl_\Phi$$
$$= \hat{R}dR + \hat{\Theta}Rd\Theta + \hat{\Phi}R\sin\Theta d\Phi$$

$$dl_R = dR$$
$$dl_\Theta = Rd\Theta$$
$$dl_\Phi = R\sin\Theta d\Phi$$

Area:

$$d\vec{s}_R = \hat{R}dl_\Theta dl_\Phi = \hat{R}R^2\sin\Theta d\Theta d\Phi$$
$$d\vec{s}_\Theta = \hat{\Theta}dl_R dl_\Phi = \hat{\Theta}R\sin\Theta dR d\Phi$$
$$d\vec{s}_\Phi = \hat{\Phi}dl_R dl_\Theta = \hat{\Phi}R dR d\Theta$$



Volume:

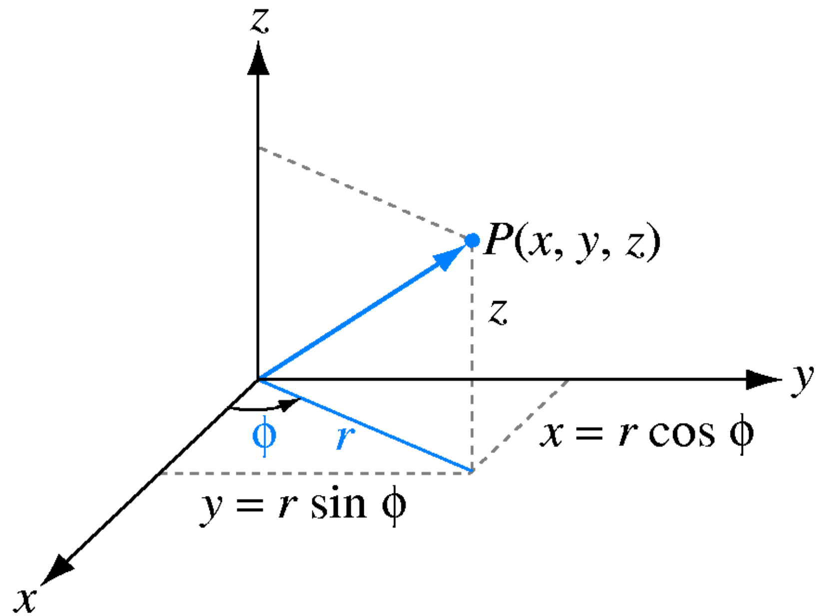
$$dv = R^2\sin\Theta dR d\Theta d\Phi$$

Cartesian to Cylindrical Transformation

$$A_r = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

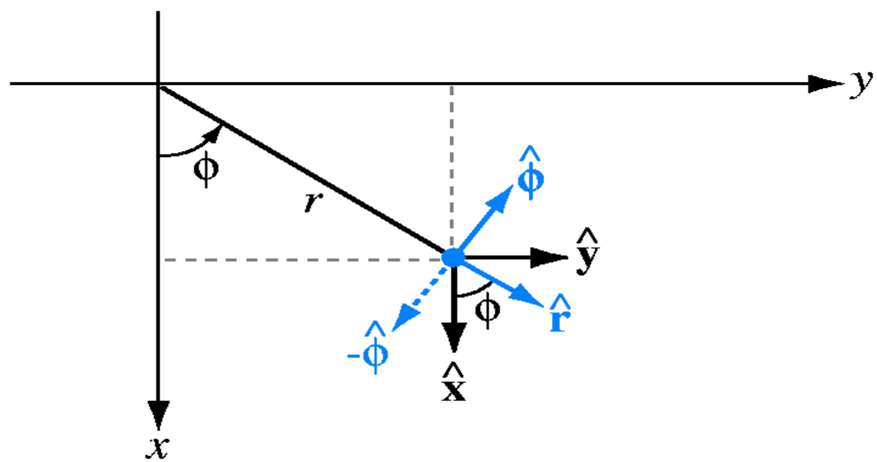
$$A_z = A_z$$



$$r = \sqrt[+]{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$



$$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$$

$$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$$\hat{z} = \hat{z}$$